UNIVERSITY OF CAMBRIDGE INTERNATIONAL EXAMINATIONS
General Certificate of Education Advanced Level

FURTHER MATHEMATICS **9231/23**

Paper 2 **May/June 2013**

3 hours

3132110842

Additional Materials: Answer Booklet/Paper
Graph Paper
List of Formulae (MF10)

READ THESE INSTRUCTIONS FIRST

If you have been given an Answer Booklet, follow the instructions on the front cover of the Booklet.
Write your Centre number, candidate number and name on all the work you hand in.
Write in dark blue or black pen.
You may use a soft pencil for any diagrams or graphs.
Do not use staples, paper clips, highlighters, glue or correction fluid.

Answer **all** the questions.
Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place in the case of angles in degrees, unless a different level of accuracy is specified in the question.
Where a numerical value is necessary, take the acceleration due to gravity to be $10\,\mathrm{m\,s^{-2}}$.
The use of a calculator is expected, where appropriate.
Results obtained solely from a graphic calculator, without supporting working or reasoning, will not receive credit.
You are reminded of the need for clear presentation in your answers.

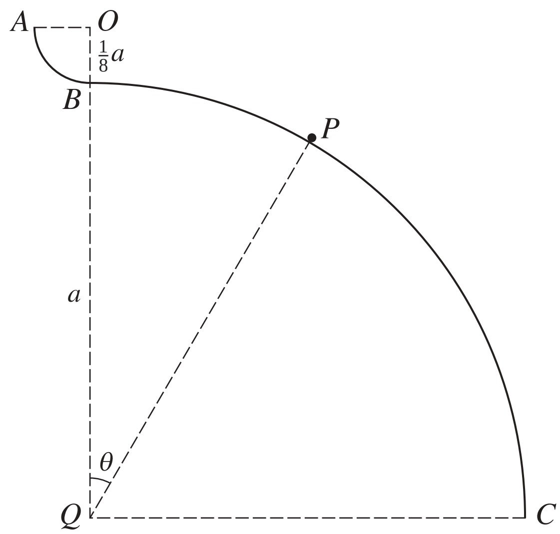
At the end of the examination, fasten all your work securely together.
The number of marks is given in brackets [ ] at the end of each question or part question.

This document consists of **5** printed pages and **3** blank pages.

UNIVERSITY of CAMBRIDGE International Examinations

1 A bullet of mass m kg is fired into a fixed vertical barrier. It enters the barrier horizontally with speed $280\,\mathrm{m\,s^{-1}}$ and emerges horizontally after 0.01 s with speed $30\,\mathrm{m\,s^{-1}}$. There is a constant horizontal resisting force of magnitude 1500 N. Find m. [4]

2



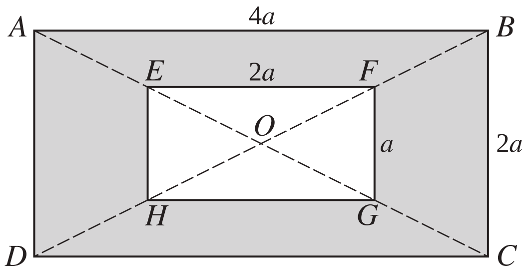
A particle P travels on a smooth surface whose vertical cross-section is in the form of two arcs of circles. The first arc AB is a quarter of a circle of radius $\frac{1}{8}a$ and centre O. The second arc BC is a quarter of a circle of radius a and centre Q. The two arcs are smoothly joined at B. The point Q is vertically below O and the two arcs are in the same vertical plane. The particle P is projected vertically downwards from A with speed u. When P is on the arc BC, angle BQP is θ (see diagram). Given that P loses contact with the surface when $\cos\theta = \frac{5}{6}$, find u in terms of a and g. [8]

3 Two uniform small smooth spheres A and B, of masses m and $2m$ respectively, and with equal radii, are at rest on a smooth horizontal surface. Sphere A is projected directly towards B with speed u, and collides with B. After this collision, sphere B collides directly with a fixed smooth vertical barrier. The total kinetic energy of the spheres after this second collision is equal to one-ninth of its value before the first collision. Given that the coefficient of restitution between B and the barrier is 0.5, find the coefficient of restitution between A and B. [9]

4 A particle P of mass m moves along part of a horizontal straight line AB. The mid-point of AB is O, and $AB = 4a$. At time t, $AP = 2a + x$. The particle P is acted on by two horizontal forces. One force has magnitude $mg\left(\frac{2a+x}{2a}\right)^{-\frac{1}{2}}$ and acts towards A; the other force has magnitude $mg\left(\frac{2a-x}{2a}\right)$ and acts towards B. Show that, provided $\frac{x}{a}$ remains small, P moves in approximate simple harmonic motion with centre O, and state the period of this motion. [5]

At time $t = 0$, P is released from rest at the point where $x = \frac{1}{20}a$. Show that the speed of P when $x = \frac{1}{40}a$ is $\frac{1}{80}\sqrt{(3ag)}$, and find the value of t when P reaches this point for the first time. [4]

5



ABCD is a uniform rectangular lamina of mass m in which $AB = 4a$ and $BC = 2a$. The lines AC and BD intersect at O. The mid-points of OA, OB, OC, OD are E, F, G, H respectively. The rectangle $EFGH$, in which $EF = 2a$ and $FG = a$, is removed from $ABCD$ (see diagram). The resulting lamina is free to rotate in a vertical plane about a fixed smooth horizontal axis through A and perpendicular to the plane of $ABCD$. Show that the moment of inertia of this lamina about the axis is $\frac{85}{16}ma^2$. [8]

The lamina hangs in equilibrium under gravity with C vertically below A. The point C is now given a speed u. Given that the lamina performs complete revolutions, show that

$$u^2 > \frac{192\sqrt{5}}{17}ag. \qquad [5]$$

6 Six pairs of values of variables x and y are measured. Draw a sketch of a possible scatter diagram of the data for each of the following cases:

(i) the product moment correlation coefficient is approximately zero;

(ii) the product moment correlation coefficient is exactly -1.

[2]

On your diagram for part **(i)**, sketch the regression line of y on x and the regression line of x on y, labelling each line. [2]

On your diagram for part **(ii)**, sketch the regression line of y on x and state its relationship to the regression line of x on y. [2]

7 Each of a random sample of 6 cyclists from a cycling club is timed over two different 10 km courses. Their times, in minutes, are recorded in the following table.

Cyclist	*A*	*B*	*C*	*D*	*E*	*F*
Course 1	18.5	17.8	19.2	22.3	16.5	20.0
Course 2	20.2	20.4	18.1	20.6	18.5	20.5

Assuming that differences in time over the two courses are normally distributed, test at the 10% significance level whether the mean times over the two courses are different. [8]

8 The number, x, of a certain type of sea shell was counted at 60 randomly chosen sites, each one metre square, along the coastline in country A. The number, y, of the same type of shell was counted at 50 randomly chosen sites, each one metre square, along the coastline in country B. The results are summarised as follows.

$$\Sigma x = 1752 \qquad \Sigma x^2 = 55\,500 \qquad \Sigma y = 1220 \qquad \Sigma y^2 = 33\,500$$

Find a 95% confidence interval for the difference between the mean number of sea shells, per square metre, on the coastlines in country A and in country B. [9]

9 A researcher records a random sample of n pairs of values of (x, y), giving the following summarised data.

$$\Sigma x = 24 \qquad \Sigma x^2 = 160 \qquad \Sigma y = 34 \qquad \Sigma y^2 = 324 \qquad \Sigma xy = 192$$

The gradient of the regression line of y on x is $-\frac{3}{4}$. Find

(i) the value of n, [3]

(ii) the equation of the regression line of x on y in the form $x = Ay + B$, where A and B are constants to be determined, [2]

(iii) the product moment correlation coefficient. [2]

Another researcher records the same data in the form (x', y'), where $x' = \dfrac{x}{k}$, $y' = \dfrac{y}{k}$ and k is a constant. Without further calculation, state the equation of the regression line of x' on y'. [2]

10 Jill and Kate are playing a game as a practice for a penalty shoot-out. They take alternate turns at kicking a football at a goal. The probability that Jill will score a goal with any kick is $\frac{1}{3}$, independently of previous outcomes. The probability that Kate will score a goal with any kick is $\frac{1}{4}$, independently of previous outcomes. Jill begins the game. If Jill is the first to score, then Kate is allowed one more kick. If Kate scores with this kick, then the game is a draw, but if she does not score then Jill wins the game. If Kate is the first to score, then she wins the game, and no further kicks are taken.

(i) Show that the probability that Jill scores on her 5th kick is $\frac{1}{48}$. [2]

(ii) Show that the probability that Kate wins the game on her nth kick is $\dfrac{1}{3 \times 2^n}$. [3]

(iii) Find the probability that Jill wins the game. [3]

(iv) Find the probability that the game is a draw. [3]

11 Answer only **one** of the following two alternatives.

EITHER

A uniform rod AB rests in limiting equilibrium in a vertical plane with the end A on rough horizontal ground and the end B against a rough vertical wall that is perpendicular to the plane of the rod. The angle between the rod and the ground is θ. The coefficient of friction between the rod and the wall is μ, and the coefficient of friction between the rod and the ground is 2μ. Show that $\tan\theta = \dfrac{1-2\mu^2}{4\mu}$. [10]

Given that $\theta \leqslant 45°$, find the set of possible values of μ. [4]

OR

A researcher is investigating the relationship between the political allegiance of university students and their childhood environment. He chooses a random sample of 100 students and finds that 60 have political allegiance to the Alliance party. He also classifies their childhood environment as rural or urban, and finds that 45 had a rural childhood. The researcher carries out a test, at the 10% significance level, on this data and finds that political allegiance is independent of childhood environment. Given that A is the number of students in the sample who both support the Alliance party and have a rural childhood, find the greatest and least possible values of A. [10]

A second random sample of size $100N$, where N is an integer, is taken from the university student population. It is found that the proportions supporting the Alliance party from urban and rural childhoods are the same as in the first sample. Given that the value of A in the first sample was 29, find the greatest possible value of N that would lead to the same conclusion (that political allegiance is independent of childhood environment) from a test, at the 10% significance level, on this second set of data. [4]

BLANK PAGE

BLANK PAGE

BLANK PAGE